Chapter 10 HW 2012

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8:11 PM

(147)

x	l_x	d_x
100	1000	200
101	800	240
102	560	252
103	308	215.6
104	92.4	92.4
105	0	0

(a) $1000A_{100} = 200\left(\frac{1}{1.035}\right) + 240\left(\frac{1}{1.04}\right)^2$

$+ 252\left(\frac{1}{1.044}\right)^3 + 215.6\left(\frac{1}{1.0475}\right)^4$

$+ 92.4\left(\frac{1}{1.05}\right)^5$

$+ \ 1 \ldots 4 (1.05)$

$= 888.06$

(b) $\ddot{a}_{100} =$

$$\frac{1000 + 800\left(\frac{1}{1.035}\right) + 560\left(\frac{1}{1.04}\right)^2 + 308\left(\frac{1}{1.044}\right)^3 + 92.4\left(\frac{1}{1.0475}\right)^4}{1000}$$

$= 2.6381$

(c) $P = \dfrac{1000\, A_{100}}{\ddot{a}_{100}} = \dfrac{888.06}{2.6381}$

$= 336.63$

(d) $f(0,1) = r_{0,1} = 0.035$

$f(1,2) = \dfrac{(1.04)^2}{1.035} - 1 = 0.045024$

$f(2,3) = \dfrac{(1.044)^3}{(1.04)^2} - 1 = 0.052046$

$f(3,4) = \dfrac{(1.0475)^4}{(1.044)^3} - 1 = 0.058071$

$f(4,5) = \dfrac{(1.05)^5}{(1.0475)^4} - 1 = 0.060060$

$(1.0425)^4$

ⓔ

$_0V = 0$

$$_1V = \frac{(_0V + P)(1 + f(0,1)) - 1000\, q_{100}}{p_{100}}$$

$$= \frac{(0 + 336.63)(1.035) - 1000(.2)}{.8}$$

$$= 185.51$$

$$_2V = \frac{(185.51 + 336.63)(1.045024) - 1000(.3)}{.7}$$

$$= 350.93$$

$$_3V = \frac{(350.93 + 336.63)(1.052046) - 1000(.45)}{.55}$$

$$= 496.98$$

$$_4V = \frac{(496.98 + 336.63)(1.058071) - 1000(.7)}{.3}$$

$$= 606.72$$

$_0V = 0$ — V

${}_5V = 0$

(f) $$f(2,5) = \left[\frac{(1+r_{0,5})^5}{(1+r_{0,2})^2}\right]^{1/3} - 1$$

$$= \left(\frac{(1.05)^5}{(1.04)^2}\right)^{1/3} - 1 = 5.672\%$$

(148)

(a) $E[D(N)] = N q_x = N(0.10)$

(b) $\mathrm{Var}[D(N)] = N \cdot q_x \cdot p_x$

$= N(0.1)(0.9) = 0.09 \cdot N$

(c) STANDARD Deviation

$= \sqrt{\mathrm{Var}[D(N)]}$

$= 0.3\sqrt{N}$

$\therefore (0.1)(E[D(N)]) = 0.3\sqrt{N}$

$(0.1)(0.1N) = 0.3\sqrt{N}$

$$\frac{.01}{0.3} = \frac{\sqrt{N}}{N}$$

$$30 = \sqrt{N}$$

$$N = 900$$

(d)

$$\frac{110 - \mu}{\sqrt{Var}} = \frac{110 - 1000(.1)}{\sqrt{.09(1000)}}$$

$$= 1.05$$

Looking up this factor in the table we get 0.8531 so

Pr(Number of Deaths > 110) =

$$1 - 0.8531 = 0.1469 = 14.69\%$$

(149)

(a) $E[D(N)] = E[E[D(N) \mid q]]$

$$= (0.25)\, E[D(N) \mid q = 0.08] +$$

$$(0.50)\, E[D(N) \mid q = 0.10] +$$

$$(0.25)\, E[D(N) \mid q = 0.12]$$

$$= (0.25)(0.08N) + (0.50)(0.10N) + (0.25)(0.12N) = 0.1N$$

(b) $Var[D(N)] = E[Var(D(N) \mid q)]$ (1)

(b) $\text{Var}[D(N)] = E[\text{Var}(D(N)|q)]$ ①

$+\text{Var}[E[D(N)|q]]$ ②

① $(.25)\,\text{Var}[D(N)|q=0.08] +$

$(0.50)\,\text{Var}[D(N)|q=0.10] +$

$(0.25)\,\text{Var}[D(N)|q=0.12] =$

$(0.25)(N(0.08)(.92)) + (0.50)(N)(.10)(.90)$

$+ (0.25)(N)(0.12)(0.88)$

$= 0.0898N$

② $(0.25)(N(.08))^2 + (0.50)(N(0.10))^2$

$+ (0.25)(N(.12))^2 - [0.1N]^2$

$= 0.0002N^2$

$\text{Var} = 0.0898N + 0.0002N^2$

(c) For 1000 policies

Expected Value $= 0.1(1000) = 100$

$\text{Var} = 0.0898(1000) + 0.0002(N^2)$

$$Var = 0.0898(1000) + 0.0002(N^2)$$

$$= 289.80$$

$$\frac{110-100}{\sqrt{289.80}} = 0.59 \Rightarrow 0.7224$$

Prob (Number of Deaths > 110)

$$= 1 - 0.7224 = 0.2776 = 27.76\%$$

150) ⓐ $P(900 + 720v + 432v^2 + 216v^3)$

$$= 160{,}000(180v + 288v^2 + 216v^3 + 216v^4)$$

$$P = 37{,}367.56$$

ⓑ $E[L_0] = (0.25)E[L_0 \mid i = 3.5\%]$

$$+ (0.50)E[L_0 \mid i = 4\%] + (0.25)E[L_0 \mid i = 4.5\%]$$

$$= (0.30)\left(A_{91}^{0.035} - 37{,}367.56\,\ddot{a}_{91}^{0.035}\right) +$$

$(0.40)(0) +$ ← since premium is calculated at 4%

$$(0.30)\left(A_{91}^{0.045} - 37{,}367.56\,\ddot{a}_{91}^{0.045}\right)$$

$$= 0.30(673.89) + 0 + (0.30)(-659.34)$$

$$= 4.37$$

$= 4.37$

$$Var[L_0] = \underbrace{E[Var[L_0 | i]]}_{①} + \underbrace{Var[E[L_0 | i]]}_{②}$$

$(0.30)\, Var[L_0 | i = 0.035] +$

$(0.40)\, Var[L_0 | i = 0.04] +$

$(0.30)\, Var[L_0 | i = 0.045]$

$$Var(L_0) = \left(100{,}000 + \frac{37{,}367.56}{d}\right)^2 \cdot \left({}^2A_x - (A_x)^2\right)$$

$i = 0.035 \Rightarrow 1{,}631{,}013{,}909.96$

$i = 0.04 \Rightarrow 1{,}636{,}305{,}642.90$

$i = 0.045 \Rightarrow 1{,}641{,}060{,}988.51$

$E[Var[L_0 | i]] = 1{,}636{,}144{,}726.70$

$Var[E[L_0 | i]] =$

$(673.89)^2(0.3) + (0)(.5) + (-659.34)^2(.3)$

$- (4.37)^2$

$- (4.37)^2$

$= 266{,}638.29$

Total Var $= 1{,}636{,}144{,}726.70 +$

$266{,}638.29$

$= 1{,}636{,}411{,}365$